Propositional Logic

↳ **Bi-conditional:** → if p and q are statements then the compound statement p if and only if q, denoted by $p \leftrightarrow q$ is called a biconditional statement and the connective if and only if the biconditional connective.

The biconditional statement $p \leftrightarrow q$ can also be written as "p is a necessary and sufficient condition for q".

$p \leftrightarrow q$ is also written as $\boxed{(p \rightarrow q) \text{ and } (q \rightarrow p)}$.

Example: →

p: He swims

q: water is warm

$p \leftrightarrow q$: He swims if and only if the water is warm.

p: I will give the party

q: I get the increment

$p \leftrightarrow q$: I will give the party if and only if when I get the increment.

Truth Table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Q) Show that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

So, $(p \leftrightarrow q) \equiv (p \to q) \wedge (q \to p)$

$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$.

Same truth values for all same input combinations.

Q) Show that $\neg(p \leftrightarrow q) \equiv (p \leftrightarrow \neg q) \equiv (\neg p \leftrightarrow q)$.

p	q	$\neg p$	$\neg q$	$p \leftrightarrow \neg q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q)$	$\neg(p \leftrightarrow q)$
T	T	F	F	F	F	T	F
T	F	F	T	T	T	F	T
F	T	T	F	T	T	F	T
F	F	T	T	F	F	T	F

All are equivalent because of same truth values.

Negation of Bi-conditional statement:

$$\neg(p \leftrightarrow q) \equiv (\neg p \leftrightarrow q) \equiv (p \leftrightarrow \neg q)$$

Q) write the negation of each of the following statements.

a) He swims if and only if the water is warm.

b) This computer Program is correct if and only if, It Produces the correct answer for all possible Sets of input data.

→ (a) He swims if and only if the water is warm.

p: He swims

q: The water is warm.

$\neg(p \leftrightarrow q) \equiv (\neg p \leftrightarrow q)$.

Negation: He does not swim if and only if the warter is warm.

or

and also $\neg(p \leftrightarrow q) \equiv (p \leftrightarrow \neg q)$

= He swims if and only if the water is not warm.

Tautology: → A tautology is a Proposition that is alwas true for all Input combinations.

~~Exam~~

Example → a> $P \vee \neg P$ b> $P \vee \neg(P \wedge q)$

↳ $P \vee \neg P$ Truth Table

P	$\neg P$	$P \vee \neg P$
T	F	T
F	T	T

→ All True

↳ $P \vee \neg(P \wedge q)$ Truth Table

P	q	$(P \wedge q)$	$\neg(P \wedge q)$	$P \vee \neg(P \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

All True for all Input Combinations.

↳ contradiction :→ A contradiction is a Proposition that is alwas false for all Input combinations.

Example : a> $P \wedge \neg P$ b> $\neg(P \vee \neg(P \wedge q))$

a>

P	$\neg P$	$P \wedge \neg P$
T	F	F
F	T	F

All false for all input combinations.

b>

P	q	$P \wedge q$	$\neg(P \wedge q)$	$(P \vee \neg(P \wedge q))$	$\neg(P \vee \neg(P \wedge q))$
T	T	T	F	T	F
T	F	F	T	T	F
F	T	F	T	T	F
F	F	F	T	T	F

All values are false for all input combi...